1. Using Armstrong’s axioms, solve the following problems:
   
   (a) (3) Given \(A \rightarrow BC\), show that \(AB \rightarrow C\).
   
   (b) (3) Given \(AB \rightarrow C\), \(DE \rightarrow F\), \(B \rightarrow D\), \(F \rightarrow C\), show that \(BE \rightarrow C\).

2. (8) Given \(F = \{E \rightarrow H, \ EH \rightarrow C, \ A \rightarrow CE, \ A \rightarrow B\}\) and \(G = \{E \rightarrow CH, \ A \rightarrow BE\}\), are \(F\) and \(G\) equivalent? Show your work.

3. Consider a relation with schema \(R(ABCDE)\) and \(F = \{BD \rightarrow C, \ AE \rightarrow D, \ C \rightarrow E, \ D \rightarrow B\}\)

   (a) (2) List all the candidate keys for this relation.
   
   (b) (4) Assuming that \(R\) is in at least 1NF, what is the highest normal form (up to and including BCNF) that \(R\) is in. Also, if you determine that \(R\) is in a normal form that is lower than BCNF then you must provide a detailed explanation as to why \(R\) is not in the next higher normal form. For example, if you determine that \(R\) is only in 2NF then you must explain, in detail using the attributes and FDs, why it is not in 3NF.
   
   (c) (3) Show that each of the candidate keys you listed above is a superkey of \(R\).

4. For the relation and decompositions given below:

   (a) (3) List all of the candidate keys of the original relation.

   (b) (3) Show whether each FD satisfies or violates 2NF, 3NF, and BCNF.

   (c) (2) State the candidate keys of each of the resulting subrelations.

   (d) (2) State the normal form (up to BCNF) of each of the subrelations. Justify your answer.

   (e) (4) Prove if the decomposition is lossless or lossy.

   (f) (4) Prove if the decomposition is FD preserving or not.

\(R(\text{DEGHJK})\) and \(F = \{GJ \rightarrow E, \ EJ \rightarrow H, \ HJ \rightarrow K, \ G \rightarrow H, \ EG \rightarrow J, \ K \rightarrow D\}\)

i) \(R_1(\text{GHJK})\) \(R_2(\text{EG})\) \(R_3(\text{DK})\) \(R_4(\text{GJ})\)

ii) \(R_1(\text{DK})\) \(R_2(\text{EGHJK})\)

iii) \(R_1(\text{EGJ})\) \(R_2(\text{DGH})\) \(R_3(\text{EHJK})\)
5. Consider the following relation and functional dependencies: $R(ABCDEFGHIJ)$ and $F = \{ B \rightarrow EG, \ EG \rightarrow AJ, CF \rightarrow BD, \ H \rightarrow AJ, \ G \rightarrow BH, \ EGI \rightarrow DH, \ G \rightarrow I \}$

(a) (15) Using a standard algorithm, decompose the relation into subrelations that are guaranteed to be in BCNF and form a lossless-join decomposition. Show your work.

(b) (15) Using a standard algorithm, decompose the relation into subrelations that are guaranteed to be in at least 3NF and form a lossless-join decomposition and a FD-preserving decomposition. Show your work.